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Gauge theory origins of supergravity causal structure

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Abstract: We discuss the gauge theory mechanisms which are responsible for the causal structure of the dual supergravity. For D-brane probes we show that the light cone structure and Killing horizons of supergravity emerge dynamically. They are associated with the appearance of new light degrees of freedom in the gauge theory, which we explicitly identify. This provides a picture of physics at the horizon of a black hole as seen by a D-brane probe.

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1. Introduction

Given the conjectured dualities between supergravity and gauge theory [1, 2], it is natural to ask how the causal structure of supergravity arises from gauge theory. In particular we would like to understand — from the gauge theory point of view — why objects in supergravity must move on trajectories that stay within their future light-cones.

This question is non-trivial because supergravity generally lives in a spacetime with more dimensions than its gauge theory dual. The extra dimensions of supergravity correspond to the moduli space of vacua of the gauge theory. The moduli space carries a Euclidean metric, with no a priori restriction on how fast an object can travel on it. So some dynamical mechanism must be present in the gauge theory to enforce causality for motion on the moduli space. The question can be sharpened by turning on a temperature. This modifies the causal structure of the supergravity background, through the appearance of a black hole with a non-degenerate horizon. How is this change in causal structure reflected by the dynamics of finite-temperature SYM?

We first consider supergravity probes, such as dilaton wavepackets in AdS_5, which map to objects that move on the base space of the gauge theory. For such probes the supergravity causal structure arises from kinematics in the SYM: causality in supergravity is related to the causal structure on the base space by the UV/IR correspondence. Then we turn to D-brane probes, made by breaking the gauge group to $\mathrm{SU}(N) \times \mathrm{U}(1)$. These probes move on the moduli space of the gauge theory. We show that the restriction on their velocities arises dynamically, through the appearance of a new light degree of freedom in the gauge theory: a pair of W particles becomes light as the velocity of a brane probe approaches the local speed of light. As a closely related phenomenon, we also discuss the limiting electric field on a D-brane probe. Finally, we turn to black holes, and show that an isolated W particle becomes massless at the horizon. This makes contact with our previous proposal [3], where we argued that a non-extremal horizon is detected by a D-brane probe as the onset of a tachyon instability [4].

To support these claims we use the SYM $\leftrightarrow$ supergravity correspondence to indirectly calculate the mass of a W in SYM, along the lines of the Wilson loop computations pioneered in [5, 6]. But in the final section of this paper we perform an explicit diagrammatic calculation of the mass of a W. The diagrammatic calculation is valid at sufficiently high temperature (at the edge of the region of correspondence with supergravity), and shows that at least in this special region these claims follow directly from the SYM itself.

2. Supergravity probes of AdS_5 causality

We begin with supergravity probes of $AdS_5 \times S^5$. This is a simple example in which the light cone structure of the supergravity arises from kinematics of the gauge theory. The metric and dilaton are

$$\begin{aligned} ds^2 &= \alpha' \left[\frac{U^2}{d_3^{1/2} e}(-dt^2 + dx_{||}^2) + \frac{d_3^{1/2} e}{U^2}\left(dU^2 + U^2 d\Omega_5^2\right)\right] \\ e^{\phi} &= \frac{g_{YM}^2}{2\pi}\,, \end{aligned} \tag{2.1}$$

where d_3 is a numerical constant and $e^2 = g_{YM}^2 N$. We need e^2, $N \gg 1$ for the supergravity to be valid. Consider an object build out of supergravity fields, at rest on the S^5, but with some velocity in the U direction. This velocity must satisfy

$$\left|\frac{dU}{dt}\right| \leq \frac{U^2}{d_3^{1/2} e} \tag{2.2}$$

in order that the object not move on a space-like trajectory.

How does such a restriction arise in the SYM? The dual $\mathrm{SU}(N)$ gauge theory lives on the boundary of AdS_5 with coordinates $(t, x_{||})$. A supergravity excitation at a radial coordinate U corresponds via holography to an excitation in the SYM with size [7, 8, 9, 10]

$$\delta x_{||} = \frac{d_3^{1/2} e}{U} \,. \tag{2.3}$$

As the supergravity probe moves in the U direction its size changes in the SYM. Translating the supergravity condition (2.2) into gauge theory terms using (2.3), one finds that causality in supergravity implies the restriction

$$\left| \frac{d\delta x_{||}}{dt} \right| < 1 \,. \tag{2.4}$$

But this is just the statement that in the gauge theory the size of an excitation can't change faster than the speed of light. An equivalent observation has been made by Susskind [11], that the time for a supergravity excitation to propagate directly across AdS space is equal to the time for a SYM excitation to travel around the boundary of AdS. Other discussions of AdS/CFT causality have been given in [12, 13, 14].

So for supergravity probes causal propagation arises as a consequence of kinematics in the SYM, via the UV/IR relationship (2.3).

3. D-brane probes of causality

We proceed to study the mechanisms which enforce causality on the motion of a p-brane probe in the near-horizon geometry of an extremal black p-brane. The dual theory is $p+1$ dimensional SYM in a certain range of couplings [15], Higgsed to $\mathrm{SU}(N) \times \mathrm{U}(1)$. We will show that causality is enforced dynamically, through the appearance of new light degrees of freedom in the gauge theory at a limiting velocity. We also discuss the closely related phenomenon of a limiting electric field.

Throughout this section we consider the extremal supergravity background ($e^2 \equiv g_{YM}^2 N$)

$$\begin{aligned} ds^2 &= \alpha' \left[\frac{U^{\frac{7-p}{2}}}{d_p^{1/2} e} (-dt^2 + dx_{||}^2) + \frac{d_p^{1/2} e}{U^{\frac{7-p}{2}}} \left(dU^2 + U^2 d\Omega_{8-p}^2 \right) \right] \\ e^{\phi} &= (2\pi)^{2-p} g_{YM}^2 \left(\frac{d_p e^2}{U^{7-p}} \right)^{\frac{3-p}{4}} \,, \end{aligned} \tag{3.1}$$

where

$$d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2} \right)$$

with B vanishing and N units of $(p+2)$-form flux on the S^{8-p}.[1]

[1] This corresponds to our use of a slightly non-standard normalization for the SYM coupling, $S_{SYM} = -\frac{1}{4g_{YM}^2} \int \mathrm{Tr} F^2 + \cdots$. We thank A. Tseytlin for pointing this out.

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3.1 Causality and the DBI action

To establish a framework for our subsequent discussion, we begin by examining the supergravity effective action for a p-brane probe. This is given by the DBI action [16, 17, 18] together with Chern-Simons couplings [19, 20].

$$S = -T_p \int e^{-\phi}\sqrt{-\det(G+B+F)} + \int C \wedge e^{B+F} \,. \tag{3.2}$$

The effects we will discuss correspond to the restriction that this action be real, $\det(G+B+F) < 0$. We would like to understand how this bound arises from SYM dynamics.

We first consider the case $B+F=0$, where the restriction simplifies to $\det G < 0$, the condition for the probe to travel inside its future lightcone. The action for the probe is

$$S = -\frac{1}{g_{YM}^2}\int d^{p+1}x\, \frac{U^{7-p}}{d_p e^2}\left(-1+\sqrt{1-\frac{d_p e^2}{U^{7-p}}\left(\dot{U}^2+U^2\dot{\Omega}^2\right)}\right) \tag{3.3}$$

and the restriction that the brane move on a time-like trajectory implies a limit on its velocity

$$\dot{U}^2 + U^2\dot{\Omega}^2 < \frac{U^{7-p}}{d_p e^2} \,. \tag{3.4}$$

In the dual SYM one considers an SU(N+1) gauge theory broken to SU(N)$\times$U(1) by a Higgs vev ρ. This gives rise to massive W's which are in the fundamental of SU(N) and charged under the U(1). We allow ρ to be time dependent.[2] In the extremal case ρ is identified with the radial coordinate U of AdS, so one has a limitation on how rapidly a Higgs vev can change in the SYM. For radial motion

$$\dot{\rho}^2 < \frac{\rho^{7-p}}{d_p e^2} \,. \tag{3.5}$$

This is somewhat surprising from the SYM point of view, since the moduli space does not carry a causal structure, and there is no a priori kinematical restriction on how fast a probe can move.

We also consider the case of a brane at rest but with a U(1) electric field ${F_0}^i$ present on its worldvolume. This is a closely related situation, since an electric field maps to velocity under T-duality. From (3.2) one sees that there is a maximal field strength

$$\sum_i \left({F_0}^i\right)^2 < 1 \,. \tag{3.6}$$

We seek to understand the SYM origin of these restrictions.

[2] A Higgs vev can't be time dependent in infinite volume, so we should think of the brane as compactified on a large torus.

All these restrictions arise because the probe action has a Born-Infeld term, which becomes singular when the velocity or electric field gets too large.[3] In string theory, the Born-Infeld action is the effective action for massless open strings. It arises from integrating out massive open string states at tree level. In general we expect a singularity of an effective action to reflect the fact that a new degree of freedom is becoming light. Such a phenomenon is well-known in the context of open string effective actions: in flat space, above a critical limiting value of the electric field, an open string with oppositely-charged ends develops a tachyon instability [21, 22]. Intuitively, when the electric field becomes larger than the string tension the string is unstable and stretches indefinitely.

The Born-Infeld action is also expected to arise as the effective action of SYM at large N [23, 24, 25]. When written in terms of the 't Hooft coupling e^2 note that the supergravity result (3.3) is $\mathcal{O}(N)$. So it should come from a sum of planar diagrams with the topology of a disk, where a W runs around the boundary of the disk and is dressed by $\mathrm{SU}(N)$ degrees of freedom that fill in the interior. The fact that the resulting effective action is non-analytic at a critical velocity or electric field suggests that some degree of freedom which was integrated out becomes light at the critical field.

It is instructive to regard this non-analyticity as a breakdown of perturbation theory. The SYM effective action has a double expansion in v^2/ρ^4 and $g^2_{\mathrm{eff}} = e^2/\rho^{3-p}$ [23]. The diagonal terms in this expansion are expected to re-sum to give the DBI action (3.3). At weak coupling, where $g^2_{\mathrm{eff}} \ll 1$ and a one-loop calculation is reliable, the perturbation series breaks down when the velocity expansion diverges, at $v^2/\rho^4 \sim 1$. This is understood as a failure of the Born-Oppenheimer approximation: W pair creation becomes possible, and gives an imaginary part to the effective action [26, 27, 28]. On the other hand, at strong effective coupling $g^2_{\mathrm{eff}} \gg 1$, it is expected that the perturbation series breaks down when the diagonal expansion parameter $g^2_{\mathrm{eff}} v^2/\rho^4$ becomes of order one: this is the point (3.5) at which the DBI action is singular. Note that at strong coupling the diagonal expansion breaks down while the velocity expansion is still good, since $v^2/\rho^4 \sim 1/g^2_{eff} \ll 1$.

As we will show in the next section, the state which becomes light as the probe velocity or electric field is increased is a pair of W's, neutral under the $\mathrm{SU}(N) \times \mathrm{U}(1)$. The mass of an isolated W remains large; it is the interaction between them that makes the pair light.

3.2 W masses at strong coupling

We want to argue that a W-pair becomes light as the velocity or electric field approaches criticality. This could in principle be studied directly in the SYM by summing planar diagrams. In section 5 we will carry out a diagrammatic calculation,

[3] The effective action is finite but non-analytic when the limiting velocity is approached. But making a Legendre transform one sees that the Hamiltonian diverges at criticality: the vacuum energy of the SYM blows up.

valid for sufficiently high temperatures, but for now we take an indirect approach, and use the correspondence with supergravity to compute the energy of a pair at strong coupling.

We do this by adopting methods introduced in [5, 6] to compute Wilson lines in large-N gauge theory. From the point of view of string theory, the worldline of a W is the boundary of a string worldsheet which has been attached to the probe D-brane. The action for the W worldline can be computed by evaluating the worldsheet action for the corresponding string. At large N and large effective coupling we can treat the worldsheet classically, ignoring fluctuations and string loop corrections.

We consider several instructive examples, which are soluble and serve to illustrate the behavior we expect in more general situations. First we examine a W pair in an electric field on a brane at rest. This provides a guide to the qualitative behavior a string worldsheet in the backgrounds of interest. Then we consider a W pair on a moving brane. For a particular family of brane trajectories we can solve for the string worldsheet, and show that the pair becomes massless exactly at criticality. Finally we study an isolated W on a brane moving along an arbitrary trajectory, and show that its mass remains large even as the velocity is increased. Although we explicitly treat the case of an AdS_5 background, the same calculations can be done for any p-brane.

3.2.1 W pair in an electric field

We seek the classical worldsheet of a fundamental string that starts and ends on a D3-brane which is located at a fixed position $U = U_f$ and has an electric field $E = F_0{}^1$ turned on in the X^1 direction. We look for a static solution to the equations of motion using the ansatz

$$\begin{aligned} X^0 &= \tau\,, \qquad -\infty < \tau < \infty \\ X^1 &= L\sigma\,, \qquad 0 \le \sigma \le \pi \\ U &= \mathrm{U}(\sigma)\,. \end{aligned}$$

This treatment is essentially identical to [5, 6]; the only new observation is that by including an electric field we can stabilize the static solution at a finite value of U_f.

The worldsheet action is Nambu-Goto plus a boundary term.[4] Evaluated on the ansatz it takes the form

$$S = -\frac{1}{2\pi}\int d\tau d\sigma \left((U')^2 + \frac{U^4L^2}{d_3e^2}\right)^{1/2} + \frac{1}{2\pi\alpha'}\int d\tau A_0|_{\sigma=0}^{\pi}\,.$$

As in [5, 6] the action is stationary when

$$\sigma = \frac{\pi}{2} + \frac{d_3^{1/2}e}{U_0L}\int_1^{\mathrm{U}(\sigma)/U_0} \frac{dy}{y^2\sqrt{y^4-1}}\,. \tag{3.7}$$

[4] A boundary term must be added for the Neumann coordinates when the gauge field is excited. But no boundary term is needed in the Dirichlet directions, for reasons discussed in [18].

Here U_0 is the minimum value of U(σ). The minimum occurs at $\sigma = \pi/2$, which gives a relation between U_0 and L.

$$\frac{\pi}{2} = \frac{d_3^{1/2} e}{U_0 L} \int_1^{U_f/U_0} \frac{dy}{y^2 \sqrt{y^4 - 1}} \,. \tag{3.8}$$

We must also impose the worldsheet boundary conditions

$$\partial_n X^\mu = F^\mu{}_\nu \partial_t X^\nu \,, \tag{3.9}$$

where ∂_n, ∂_t are the unit normal and tangential derivatives

$$\begin{aligned} \partial_n &= \frac{\left(d_3 e^2\right)^{1/4} U_f}{\sqrt{U_f^4 L^2 + d_3 e^2 (U')^2}} \partial_\sigma \\ \partial_t &= \frac{\left(d_3 e^2\right)^{1/4}}{U_f} \partial_\tau \,. \end{aligned} \tag{3.10}$$

The boundary conditions are satisfied when $U_0 = E^{1/2} U_f$. Note that as we increase the electric field U_0 approaches U_f, so the string stays closer to the 3-brane. But at the same time by (3.8) the separation $L\pi$ between the two W particles decreases. Note that L goes to zero at the critical field $E = 1$; this may be an artifact of our static ansatz.

3.2.2 W pair at criticality

Consider a brane moving along a trajectory $U = \Phi(X^0)$, and make the following ansatz for a string worldsheet.

$$\begin{aligned} X^0 &= \tau \\ X^1 &= L\sigma \\ U &= \Phi(\tau) \,. \end{aligned}$$

This describes a segment of string lying entirely within the the brane worldvolume. The equations of motion for the segment of string are satisfied provided the brane trajectory obeys

$$\ddot{\Phi} - \frac{4\dot{\Phi}^2}{\Phi} + \frac{2\Phi^3}{d_3 e^2} = 0 \,.$$

This is not the same as the equation of motion of a freely-falling brane in AdS, so in general external forces must act on the brane to get it to follow such a trajectory. But even a brane subject to external forces must obey the restrictions of causality. And the above ansatz is sufficient to illustrate the effects we are interested in, since a particular solution is $\Phi(X^0) = d_3^{1/2} e / X^0$, which corresponds to a brane moving radially at the local speed of light.[5]

[5]This is not physically possible in supergravity, of course, but we are only using this solution to understand how the same bound arises in SYM.

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We must also satisfy the boundary conditions (3.9) at the string endpoints. This can be done by turning on a worldvolume electric field $E = {F_0}^1$ which satisfies

$$E^2 = 1 - \frac{d_3 e^2 \dot{\Phi}^2}{\Phi^4} \,. \tag{3.11}$$

When this condition is obeyed the system is at criticality. Note that the length of the string in the X^1 direction $L\pi$ remains arbitrary. Also note that when the velocity of the brane equals the local speed of light the electric field necessary to stabilize the string endpoints vanishes.

Evaluating the action for this configuration we find

$$\begin{aligned} S_{bulk} &= -\frac{L}{2}\int d\tau \sqrt{\frac{\Phi^4}{d_3 e^2} - \dot{\Phi}^2} \\ S_{boundary} &= \frac{L}{2}\int d\tau \frac{\Phi^2}{d_3^{1/2} e} E \,. \end{aligned}$$

When the boundary conditions (3.11) are satisfied the bulk and boundary actions exactly cancel, showing that a W pair becomes massless at arbitrary separation in a critical field.

3.2.3 Mass of an isolated W

As we have seen, a W pair becomes massless when the electric field and velocity become critical. This could occur in the gauge theory for two distinct reasons: either the mass of an individual W particle goes to zero and the interactions are small, or the mass of a single W remains finite but a large interaction potential cancels the rest energy of the pair.

To decide between these possibilities we study a single W in isolation. This is easiest in the velocity case, with vanishing electric field. An isolated W can be obtained from a W pair by sending the separation $L \to \infty$ while holding the velocity fixed. As can be seen in the example of section 3.2.1, at large L the worldsheet will tend to adopt a configuration in which the string stretches from the brane close to the horizon at $U = 0$ and then travels along the horizon before returning back out to the brane.[6]

This means that we can compute the mass of an isolated W by studying the worldsheet of a string that stretches straight from the brane to the horizon. Such a solution is easy to obtain: for a brane moving on an arbitrary trajectory $U = \Phi(X^0)$ the string worldsheet is

$$X^0(\tau,\sigma) = \tau \,, \qquad \mathrm{U}(\tau,\sigma) = \frac{1}{\pi}\,\sigma\,\Phi(\tau) \,.$$

[6]For an extremal background we expect the classical string worldsheet to change smoothly as L increases, but at finite temperature the minimal-action worldsheet changes discontinuously at a finite value of L [29, 30].

Evaluating the Nambu-Goto action for this worldsheet one finds

$$S = -\frac{1}{2\pi} \int dX^0 \, \Phi(X^0) \, .$$

The W is at rest in the gauge theory, so we can directly read off its (time-dependent) mass: $m_W = \frac{1}{2\pi}\Phi$.

Note that m_W is given in terms of the instantaneous Higgs vev by the usual formula, without any corrections involving the velocity. In particular m_W remains finite as the velocity of the brane approaches the speed of light. This means that the phenomenon discussed above, of a W pair at some fixed separation becoming light at the critical velocity, must be understood as due to a large interaction energy between the two W's, which cancels their rest energy in the limit of critical velocity.

Given that W pairs become light as the velocity or electric field is increased, it is natural to wonder if large amounts of pair production will take place. We feel that pair production remains highly suppressed below criticality, for reasons that are simplest to explain for electric fields. Note that although the energy of a W pair becomes small, the mass of an isolated W remains large, so the pair is not completely free to separate in the electric field. Indeed the static classical solution (3.7) suggests that as the electric field is increased the two W particles are forced to move closer together. This reduces their dipole moment, which makes them very weakly coupled to the electric field. From the point of view of string theory, apart from the fact that pair production is suppressed by $g_s \to 0$, the W pair corresponds to a neutral string, which further suppresses pair production [26]. So we do not expect much pair production to take place as long as the field is below critical.

Pair production should set in above the critical field. For classical strings this happens abruptly, as soon as the electric field exceeds the critical value. But it would be interesting to understand whether quantum effects can smooth out the transition. This is an issue of $1/N$ corrections to the gauge theory. The classical computations we have performed are only valid in the strict $N \to \infty$ limit. At finite N the SU(N) degrees of freedom can fluctuate, and it seems reasonable that this corresponds to the light-cone fluctuations expected in quantum gravity.

4. D-brane probes of horizons

It is interesting to extend our discussion of causality to include finite temperature effects. On the supergravity side, finite temperature modifies the causal structure of the spacetime, through the formation of a black hole with a non-degenerate horizon. The presence of this horizon must somehow be reflected in the finite-temperature dynamics of SYM.

To be more precise, we wish to address the following question. The dual gauge theory selects a preferred set of coordinates for supergravity. The time coordinate of the SYM corresponds to a timelike Killing vector of the supergravity background. In supergravity this Killing vector can become null, at so-called Killing horizons. For example, this always happens at the event horizon of a stationary black hole [31], and it also happens in AdS_5 at $U = 0$ (which is not a true event horizon). What is the signature of a Killing horizon in the gauge theory?

To study this we place a D-brane probe at some Higgs vev ρ in the gauge theory. Our proposal is that when the value of the Higgs vev corresponds to the position of a Killing horizon, the mass of a W goes to zero in the gauge theory. Note that we are talking about a single isolated W particle, not the W-pair discussed in the previous section.

This proposal originated in our earlier work [3], where we studied the way in which a black hole absorbs and thermalizes a D-brane probe. We argued that the W (mass)^2 matrix has a zero eigenvalue when the Higgs vev takes the value $\rho = \rho_0$ which corresponds to the position of the horizon.[7] For non-extremal black holes we argued that there is a tachyon instability for $\rho < \rho_0$. The tachyon instability gives an imaginary part to the probe effective potential, which causes absorption, and also makes it possible for the probe to rapidly thermalize with the black hole. For extremal black holes a W goes massless but no tachyon develops, and the absorption mechanism is more plausibly string pair creation [3].

We first consider the case of degenerate horizons (Killing horizons with zero surface gravity[8]). For example, consider the near-horizon geometry of an extremal black p-brane (3.1). The timelike Killing vector ∂_t becomes null at $U = 0$. For a Dp-brane probe this point indeed corresponds to the origin of moduli space, where the W's become massless. Another example is a D-string probe of an extremal D1-D5-momentum black hole [32, 33]. Again the timelike Killing vector becomes null at the point which corresponds to the origin of moduli space, where new massless particles appear. These extremal black holes are supersymmetric, so no tachyon instability can develop, which means that the thermalization rate for infalling matter is very slow [3]. But this is consistent with the fact that these degenerate horizons have zero temperature.

Let us now consider the non-degenerate Killing horizons which arise for non-extremal black branes. For simplicity we treat the near-horizon geometry of a ten-dimensional 0-brane black hole, although the same discussion could be given for any p-brane charge [15]. The string-frame metric and dilaton are

$$\begin{aligned} ds'^2 &= \alpha' \left[-h(U) \frac{U^{7/2}}{d_0^{1/2} e} dt^2 + h^{-1}(U) \frac{d_0^{1/2} e}{U^{7/2}} dU^2 + \frac{d_0^{1/2} e}{U^{3/2}} d\Omega_8^2 \right] \\ e^{\phi} &= 4\pi^2 g_{YM}^2 \left(\frac{d_0 e^2}{U^7} \right)^{3/4} , \end{aligned} \tag{4.1}$$

[7] At finite N this is better thought of as the position of the stretched horizon (see the discussion section).

[8] but possibly non-zero area

where $h(U) = 1 - U_0^7/U^7$. The horizon is located at $U = U_0$.

By evaluating the action (3.2) in the background (4.1) we find that the effective action for a 0-brane probe is

$$S = -\frac{1}{g_{YM}^2}\int dt \frac{U^7}{d_0 e^2}\left(-1 + \sqrt{h(U) - \frac{\dot{U}^2 d_0 e^2}{h(U)U^7}}\right). \tag{4.2}$$

Thus there is a limit on the radial velocity of the probe

$$\frac{dU}{dt} < \frac{h(U)U^{7/2}}{d_0^{1/2}e}.$$

The simple identification between the U coordinate of supergravity and the Higgs vev ρ of the gauge theory no longer holds when the black hole has finite temperature. Rather, to get the kinetic terms to agree, one must set [34, 35]

$$\frac{d\rho}{\rho} = \frac{dU}{U\sqrt{h}}. \tag{4.3}$$

This gives the relation

$$\rho^{7/2} = \frac{1}{2}\left(U^{7/2} + \sqrt{U^7 - U_0^7}\right). \tag{4.4}$$

At extremality this reduces to the expected result $\rho = U$, and we see that an extremal horizon indeed maps to the origin of moduli space. But away from extremality the horizon corresponds to a non-zero Higgs vev $\rho_0 = \rho(U_0) \sim U_0$.

Note that the interior of the horizon $U < U_0$ in supergravity corresponds to imaginary Higgs vevs in the gauge theory, so the interior of the event horizon cannot be directly described by the gauge theory: the gauge theory provides an outside observer's description of the black hole. The degrees of freedom which are associated with the black hole are indirectly encoded in the gauge theory. As we discuss in the conclusions, the Higgs vev ρ_0 should really be associated with the stretched horizon of the black hole. Thus the gauge theory degrees of freedom at $\rho < \rho_0$ provide an effective description of the black hole degrees of freedom seen from the point of view of an outside observer, a description in the spirit of the membrane paradigm [36].

The restriction on velocity translates by (4.4) into a restriction on the rate of change of the Higgs vev

$$\frac{d\rho}{dt} < \frac{\rho^{7/2}}{d_0^{1/2}e}\frac{(1 - \frac{\rho_0^7}{\rho^7})}{(1 + \frac{\rho_0^7}{\rho^7})^{2/7}}. \tag{4.5}$$

Away from the horizon the physics of this bound is essentially the same as in the previous section, with a W-pair becoming massless due to interactions when the velocity approaches its limiting value.

But note that even for zero velocity the effective action (4.2) is singular at the horizon, where $h(U)$ vanishes.[9] This non-analyticity signals the breakdown of the Born-Oppenheimer approximation. But unlike the situation in section 3, the breakdown is now due to an isolated W becoming massless. To show this, we proceed as in section 3.2.3, and calculate the mass of an isolated W from the worldsheet action for an elementary string stretched between the probe and the horizon. This is identical to the calculations performed in [29, 30] in the context of finite-temperature Wilson lines. For a brane located at $U = \Phi(X^0)$ this gives the W mass $m_W = (\Phi - U_0)/2\pi$. We see that a single W indeed becomes massless at the horizon. Extrapolation to $\Phi < U_0$ would give an imaginary mass, since U becomes a timelike coordinate inside the horizon.

5. Causality in M(atrix) theory

For non-conformal p-branes the supergravity background (3.1) is only valid for a limited range of the radial coordinate [15]. In particular, for a system of D0-branes, the description in terms of ten-dimensional supergravity breaks down at short distances, and must be replaced by M-theory. We now briefly discuss causality in this regime.

A minor point is that M(atrix) theory [1] has a decoupled U(1) sector describing center of mass motion. In the c.m. sector the theory is free and there is no limit on how rapidly a scalar field can change. But this is consistent with the fact that M(atrix) theory is a light-front description of M-theory. In light front coordinates the requirement of causality[10]

$$\dot{x}_\perp^2 < 2\dot{x}^-$$

puts no restriction on the transverse c.m. velocity: for any $\dot{x}_\perp$ this inequality can be satisfied simply by making $\dot{x}^-$ sufficiently large.

A restriction does arise, however, on the transverse relative velocity. This is because relative motion is governed by the smeared Aichelburg-Sexl metric in eleven dimensions [23]

$$ds^2 = -dx^+ dx^- + dx_\perp^2 + \frac{15N}{2R^2M^9r^7}\left(dx^-\right)^2 . \tag{5.1}$$

Causality then requires

$$\dot{x}_\perp^2 < 2\dot{x}^- - \frac{15N}{2R^2M^9r^7}\left(\dot{x}^-\right)^2 .$$

This can only be satisfied for some $\dot{x}^-$ provided that the transverse relative velocity is bounded,

$$\dot{x}_\perp^2 < \frac{2R^2M^9r^7}{15N}$$

[9] The effective action as a function of ρ looks analytic, but $\rho(U)$ is non-analytic at the horizon.
[10] An overdot denotes ∂_τ, where $\tau = \frac{1}{2}x^+ = \frac{1}{2}\left(t + x^{11}\right)$ as in [23].

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or equivalently

$$\dot{U}^2 + U^2\dot{\Omega}^2 < \frac{U^7}{240\pi^5 e^2}\,, \tag{5.2}$$

where we have used $RM^3 = 1/2\pi\alpha'$ and $R = (2\pi\alpha')^2 g_{YM}^2$. This agrees precisely with the bound (3.4) extracted from the ten dimensional supergravity background.

This agreement isn't surprising, since the classical supergravity backgrounds are so closely related. But it does suggest that W-pairs become massless in the gauge theory at the critical velocity (5.2), even in the M(atrix) theory regime.

6. Direct calculation of the W mass

In this section we discuss the direct evaluation of the W mass for a system of D0-branes at a particular temperature, and show that it agrees with the supergravity result for a black hole of the same temperature. The supergravity is only valid up to black hole temperatures of order $T_c \sim e^{2/3}$, because at this temperature the curvature at the horizon becomes of order one in string units [15]. Above T_c the gauge theory is weakly coupled. Therefore at $T \sim T_c$ we should be able to qualitatively match perturbative SYM calculations onto supergravity calculations.

We wish to compute the thermal partition function of 0+1 dimensional SYM, with Euclidean action [37, 1] ($e^2 = g_{\rm YM}^2 N$)

$$S_{SYM} = \frac{1}{g_{YM}^2}\int_0^\beta d\tau\ \mathrm{Tr}\Big\{\frac{1}{2}\partial_\tau X^i \partial_\tau X^i - \frac{1}{4}[X^i,X^j][X^i,X^j] + \\ +\Psi_a\partial_\tau\Psi_a + \Psi_a\gamma^i_{ab}[X^i,\Psi_b]\Big\}\,. \tag{6.1}$$

We work in the high temperature regime $T > T_c$. At high temperatures the gauge theory is weakly coupled, so the naive expectation is that the partition function can be obtained from the tree-level spectrum of massless bosons and fermions. This logic works for the fermions but fails for the bosons, because higher loop corrections, although suppressed by powers of the coupling, are in fact infrared divergent due to the bosonic zero modes.[11]

These infrared divergences are cured through the generation of a thermal mass for the bosons. To obtain the mass gap we must re-sum part of the perturbation series. This can be done as follows. As we argue below, at high temperatures the fermions make a negligible contribution to the mass gap. So we temporarily ignore them, and approximate the bosonic sector of the SYM action (6.1) by the Gaussian action

$$S_0 = \sum_{l=-\infty}^{\infty} \frac{1}{2\sigma_l^2}\mathrm{Tr}\left(X_l^i X_{-l}^i\right). \tag{6.2}$$

[11] At finite temperature the fermions are antiperiodic and do not have zero modes.

Here l labels Fourier modes in the Euclidean time direction. To fix the parameters σ_l^2 we make a large-N variational approximation [38]. This gives a gap equation for the effective propagator

$$\frac{1}{\sigma_l^2} = \frac{1}{g_{\rm YM}^2}\left(\frac{2\pi l}{\beta}\right)^2 + \frac{16N}{g_{\rm YM}^2\beta}\sum_m \sigma_m^2\,. \tag{6.3}$$

By scaling this equation one can show that the dimensionless effective propagator at large N, namely $\widetilde{\sigma}_l^2 = N\sigma_l^2/e^{2/3}$, only depends on the dimensionless effective temperature $\widetilde{T} = T/e^{2/3}$.

The second term on the right hand side of the gap equation is a thermal mass for the bose fields. In fact this gap equation takes into account all leading high-temperature corrections to the bose propagator. In writing the gap equation we have ignored corrections to the propagator involving a single fermion loop, as well as multiloop corrections. But the fermion loop is negligible at high temperatures (it vanishes in the high temperature limit, due to the antiperiodic boundary conditions for the fermions). And multiloop corrections to the propagator are small at high temperatures, where the theory is weakly coupled.[12]

To solve the gap equation we introduce the 'size' of the thermal state, defined by the range of eigenvalues of the matrices X^i.

$$\begin{aligned} R_{\rm rms}^2 &\equiv \frac{1}{N}\left\langle {\rm Tr}\,\left(X^i(\tau)\right)^2\right\rangle \qquad\qquad \text{(no sum on } i\text{)} \\ &= \frac{N}{\beta}\sum_l \sigma_l^2\,. \end{aligned}$$

The gap equation gives a consistency condition which determines $R_{\rm rms}$.

$$R_{\rm rms}^3 = \frac{e^2}{8\tanh\left(2\beta R_{\rm rms}\right)}\,. \tag{6.4}$$

In this approximation the SYM consists of $9N^2$ bosonic harmonic oscillators with frequency $4R_{\rm rms}$, plus $8N^2$ decoupled massless fermions, so the SYM free energy at high temperatures is given by[13]

$$\beta F_{SYM} \approx 9N^2\log[2\sinh(2\beta R_{\rm rms})] - 8N^2\log 2\,. \tag{6.5}$$

Thus at high temperatures ($T \gg e^{2/3}$) we have the approximate size, energy and entropy

$$R_{\rm rms} \approx \frac{1}{2}e^{1/2}T^{1/4}$$

[12]Once the thermal mass has been taken into account multiloop corrections truly are suppressed at weak coupling.

[13]There are also ghosts associated with the choice of axial gauge $A_0 = 0$, but their contribution to the partition function is just $\det{}'\,\partial_\tau = 1$.

$$
\begin{aligned}
E_{SYM} &\approx \frac{27}{4} N^2 T \\
S_{SYM} &\approx N^2 \left(\text{const.} + \frac{27}{4} \log\left(\frac{T}{e^{2/3}}\right).\right)
\end{aligned} \tag{6.6}
$$

These are the standard results for perturbative SYM in this temperature regime.

The correct results in the low temperature regime $T \ll e^{2/3}$ can be extracted from the dual supergravity background (4.1). One finds that the horizon radius, energy and entropy are given by [15]:

$$
\begin{aligned}
U_0 &\sim e^{2/3} \left(\frac{T}{e^{2/3}}\right)^{2/5} \\
E_{SUGRA} &\sim N^2 e^{2/3} \left(\frac{T}{e^{2/3}}\right)^{14/5} \\
S_{SUGRA} &\sim N^2 \left(\frac{T}{e^{2/3}}\right)^{9/5} .
\end{aligned} \tag{6.7}
$$

We see that the supergravity results for the energy and entropy agree with the perturbative SYM results (6.6) at $T \sim e^{2/3}$. Moreover, the Higgs vev ρ_0 at which a probe 0-brane develops a tachyon instability in the Gaussian ensemble (6.2) can be computed, and one finds that [3]

$$
\rho_0 \left(T = e^{2/3}\right) \sim e^{2/3} . \tag{6.8}
$$

This agrees with the supergravity result that $U_0|_{T=e^{2/3}} \sim e^{2/3}$. Thus on the edge of validity of the supergravity description one can apply a Gaussian approximation to the SYM, and one finds that the horizon of the black hole indeed coincides with the onset of a tachyon instability for a 0-brane probe.

Directly analyzing the SYM at low temperatures is a difficult problem. But the bosonic sector of the gauge theory is easier to study: the Gaussian approximation (6.2), (6.3) is a good description of bosonic large-N Yang-Mills quantum mechanics at any temperature.[14] Thus for the bosonic theory we have

$$
\beta F_{\text{bose}} \approx 9 N^2 \log[2 \sinh(2 \beta R_{\text{rms}})] .
$$

In particular, in the low temperature regime ($T \ll e^{2/3}$)

$$
\begin{aligned}
R_{\text{rms}} &\approx \frac{1}{2} e^{2/3} \\
E_{\text{bose}} &\approx 9 N^2 e^{2/3} \\
S_{\text{bose}} &\sim N^2 \gamma(T)
\end{aligned} \tag{6.9}
$$

where $\gamma(0) = 0$ and $\gamma(e^{2/3}) = 1$. The energy is just the ground state energy of the bosonic harmonic oscillators. Notice that even at zero temperature the ground state

[14] One can calculate the leading corrections to the Gaussian approximation for the bosonic partition function and show that they are small.

fluctuations $\sim R_{\rm rms}$ in the bosonic theory are very large. In fact they extend out to the edge of the region $\sim e^{2/3}$ in which (in the supersymmetric case) supergravity is valid.

At temperatures below $e^{2/3}$ the fermions in the SYM are crucial for canceling the enormous ground state energy of the bosons which appears in (6.9). While inclusion of the fermions will lower the ground state energy it cannot significantly alter $\langle \mathrm{Tr}\, X^2 \rangle$, which continues to correspond to the size of the entire region in which supergravity is valid. So generically all the interesting supergravity physics, and in particular the horizon radius U_0, lies deep inside the region of the bosonic ground state fluctuations [11, 39].

7. Conclusions and discussion

We have analyzed the emergence of causal structure in supergravity in view of the SYM $\leftrightarrow$ supergravity correspondence. We have shown that for D-brane probes the supergravity restriction, that a probe travel slower than the local speed of light, arises dynamically in the gauge theory, through the appearance of a light composite state of two W's. The same phenomenon is responsible for the limitation on electric field. We have also confirmed that a Killing horizon in supergravity is associated with a W becoming massless, and that with non-zero surface gravity the W may further become tachyonic. It is amusing to see that whenever a D-brane probe tries to move on a lightlike path, a massless mode appears in the SYM. This gives a coherent and simple picture of the origin of supergravity causal structure for D-brane probes.

We would like to comment on the picture of black hole physics that seems to arise. It has been argued [40, 41] that there should be a unitary description of black hole formation and evaporation from the point of view of an external observer. For this to happen the semiclassical approximation to quantum gravity (i.e. matter fields propagating on a fixed background) should break down near the black hole horizon even though the curvature there is small. The black hole should have a "stretched horizon", at a radius slightly larger than the radius of the event horizon. Hawking radiation emerges from the stretched horizon which is endowed with the thermal properties of the black hole. It can be shown [42] that the semiclassical approximation breaks down if one uses spacetime foliations associated with outside observers. In this scheme the stretched horizon arises naturally as the locus of points where the semiclassical approximation breaks down. It was also argued that the description of physics as seen by observers falling into the black hole is complementary to the description of the black hole as seen by the external observers.

In the supergravity $\leftrightarrow$ SYM correspondence one identifies the SYM time coordinate with the timelike Killing coordinate outside the black hole horizon. This means that the SYM description of a black hole state corresponds to an outside observer's description of black hole physics.[15] In classical supergravity, corresponding to leading $1/N$ effects in the SYM, the stretched horizon and event horizon cannot be distinguished. This is the limit we have considered in this paper. Subleading corrections, however, do distinguish them. At finite N the thermodynamic limit is replaced by statistical mechanics, and fluctuations can occur. In particular the value of the tachyon instability radius will fluctuate. It seems natural to us to associate the outer limit of the tachyon instability with the stretched horizon. This is why a probe will thermalize at the stretched horizon, where the semiclassical approximation breaks down. As the gauge theory develops a massless mode at the stretched horizon, the IR/UV correspondence suggests that physics there is influenced by quantum gravity effects.

Below the Higgs vev corresponding to the stretched horizon, the gauge theory has no space time interpretation. Instead the probe has entered a region where the non-Abelian degrees of freedom control the physics. Thus for the outside observer the region behind the stretched horizon has no spacetime interpretation, unlike the situation for an infalling observer.

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